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A Method of Exchanging Digital Data

The invention relates to digital data exchange and electronic commerce, and in particular, to a method of fair and efficient exchange of digital data between potential distrustful parties over a digital communication channel.

An important issue in information processing and electronic commerce is how to exchange non-repudiation information between two potentially distrustful parties in a secure and fair manner. An example of this is the electronic contract signing problem where two parties are physically apart and negotiate a contract in the form of digital document over a communication network. The contract is considered legally binding if the two parties have each other's digital signatures on the digital document. The two parties need to execute a fair exchange protocol to obtain each other's digital signatures. Other applications of fair exchange protocols include certified electronic mail delivery and electronic auctioning over internet.

Fair exchange has been studied for some time in the context of "simultaneous secret exchange" or "gradual secret releasing", see for examples, S. Even, O. Goldreich, and A. Lempel, "A randomized protocol for signing contracts", Communications of the ACM, vol. 28, pp. 637-647, June 1985; also see T. Okamoto and K. Ohta, "How to simultaneously exchange secrets by general assumptions", Proceedings of the 2nd ACM Conference on Computer

and Communications Security, pp. 184-192, Fairfax, Virginia, November 1994. In simultaneous secret exchange schemes, it is assumed that two parties A and B each possess a secret a and b, respectively, where a and b are n bit strings. Further it is assumed that both secrets represent some value to the other party and that they are willing to trade the secrets with each other. A simultaneous secret exchange process is typically carried out as following. First, A and B exchange  $f(a)$  and  $g(b)$  for some predefined functions  $f()$  and  $g()$ , with the property that A can not get b from  $g(b)$  and B can not recover a from  $f(a)$ . Then, A and B release a and b bit-by-bit. For such a protocol to be useful, it must satisfy the following two requirements: correctness -- the correctness of each bit given must be checked by each receiver to ensure that his/her secret has not being traded for garbage; and fairness -- the computational effort required from the parties to obtain each other's remaining secret should be approximately equal at any stage during the execution of the protocol. Note that the above fairness definition based on equal computational complexity makes sense only if the two parties have equal computing power, an often unrealistic and undesirable assumption. Another drawback of the above scheme is that the execution of the scheme requires many rounds of interactions between the two parties.

The other approach in fair exchange is using an on-line trusted third party (TTP), see for examples, J. Zhou and D. Gollmann, "A fair non-repudiation protocol", Proceedings of the 1996 IEEE

Symposium on Security and Privacy", IEEE Computer Press, pp. 55-61, Oakland, CA; R. H. Deng, L. Gong, A. A. Lazar, and W. Wang, "Practical protocols for certified electronic mail", Journal of Network and Systems Management, vil. 4, no. 3, pp. 279-297, 1996. In on-line TTP based protocols, the TTP acts as a middleman. A and B forward their messages/signatures to the TTP. The TTP first checks the validity of the received signatures and then relays them to the respective parties. The major drawback of this approach is that the TTP is always involved in the exchange even if the parties are honest and no fault occurs; therefore, the on-line TTP is both a computational bottleneck and a communications bottleneck. To avoid such bottlenecks, a more novel approach is to use protocols with an off-line TTP. That is, the TTP does not get involved in the normal or exceptionless case, it gets involved only in the presence of faults or in the case of dishonest parties who do not follow the protocols.

To our knowledge, the only fair exchange protocols using off-line TTP are given by N. Asokan, M. Schunter, and M. Waidner, "Optimistic protocols for fair exchange", Proceedings of the 4th ACM Conference on Computer and Communications Security, Zurich, April 1997. However, these protocols achieve fairness only if the TTP can undo a transfer of an item or it is able to produce a replacement for it; otherwise, a misbehaving party may get other party's data and refuse to send his data to the other party. When this happens, all the TTP can do in the above mentioned protocols is to issue affidavits

attesting to what happened during the exchange. However, such affidavits may be useless in the internet environment where the cheating party may disappeared easily and the damage to the honest party may not be revocable.

In accordance with the present invention, a method of exchanging digital data between a first party, having a unique first digital data, and a second party, having a unique second digital data, over a communication link, the method comprising the steps of:

(a) the first party encrypting the first digital data and generating an authentication certificate, the authentication certificate authenticating that the encrypted first digital data is an encryption of the first digital data, and sending the encrypted first digital data and the authentication certificate to the second party;

(b) the second party verifying that the encrypted first digital data is an encryption of the first digital data using the authentication certificate, and the second party sending the second digital data to the first party if the verification is positive;

(c) the first party verifying that the second digital data is valid, and if the verification is positive the first party accepts the second digital data and sends the unencrypted first digital data to the second party;

(e) the third party decrypting the encrypted first digital data to obtain the first digital data, verifying that the first and the second digital data are valid and, if both the first and the second digital data are verified as valid, sending the first digital data to the second party and the second digital data to the first party.

The invention provides a method of exchanging digital data between distrustful parties over a communication link, and has the advantages of 1) using an off-line trusted third party (TTP), i.e., TTP does not take part in the exchange unless one of the exchanging parties behaves improperly; 2) being efficient in communications, only three message exchanges are required in the normal situation; and 3) achieving fairness, i.e., either A and B obtain each other's data or no party receives anything useful, and no loss is incurred to a party no matter how maliciously the other party behaves during the exchange.

Fairness is only achieved if the exchange protocol possesses a so called loss-preventing property. Loss-preventing means that

no loss is incurred to a party no matter how improperly the other party performs. More specifically, an exchange protocol achieves true fairness if it guarantees that either both parties obtain each other's signatures or none of them get anything. The exchange systems presented in this invention are the first which achieve true fairness with off-line TTP.

A new cryptographic primitive, called the Certificate of Encrypted Message Being a Signature (CEMBS) is also invented here. The CEMBS is used to prove that an encrypted message is a certain party's signature on a file without revealing the actual signature.

Examples of a method of exchanging digital data in accordance with the invention will now be described with reference to the accompanying drawings, in which:

Figure 1 shows the steps of fair exchange digital signatures on a common file;

Figure 2 shows the steps of fair exchange of a file and a digital signature on a one-way hash of the file;

Figure 3 illustrates the flow diagram of the first Signature-Ciphertext-CEMBS-Generation Program (SCCGP) used in the preferred embodiment of the present invention; and,

Figure 4 shows the flow diagram of the second Signature-Ciphertext-CEMBS-Generation Program (SCCGP) used in the preferred embodiment of the present invention.

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The parties involved in the protocols and some of the notations used in the description of the examples are as follows.

#### Notations related to public key encryption scheme

$P$  : a public key encryption scheme  
 $P_{\text{encr}}$  : encryption algorithm of  $P$   
 $P_{\text{decr}}$  : decryption algorithm of  $P$   
 $PK$  : a public key in  $P$   
 $SK$  : the private key corresponding to  $PK$   
 $P_{\text{encr}}(PK, m)$  : encryption output (i. e., ciphertext) of a plaintext  $m$  using  $PK$   
 $P_{\text{decr}}(SK, c)$  : decryption output (i. e., plaintext) of a ciphertext  $c$  using  $SK$

#### Notations related to digital signature schemes

$S$  : a digital signature scheme  
 $S_{\text{sign}}$  : signing algorithm of  $S$   
 $S_{\text{veri}}$  : verifying algorithm of  $S$   
 $sk$  : a private (or signing) key in  $S$   
 $pk$  : the public (or verifying) key corresponding to  $sk$   
 $S_{\text{sign}}(sk, m)$  : signature on a plaintext  $m$  under private key  $sk$   
 $S_{\text{veri}}(pk, \text{sign}, m)$  : verification of a signature  $\text{sign}$  on a message  $m$  using public key  $pk$ ; it outputs  
 yes if the signature is valid and no

otherwise

## Mathematics notations

$a^b$  : a raised to the bth power

$a||b$  : the concatenation of  $a$  and  $b$

$\mathbb{Z}_p$  : the set of  $p$  integers  $\{0, 1, 2, \dots, p-2, p-1\}$

$\mathbb{Z}_p^*$  : the subset of integers in  $\mathbb{Z}_p$  which are relatively prime to  $p$

There are three generic parties in a fair-exchange system,

## Parties involved

A : a party involved in a fair exchange. It has a pair of public/private keys  $pk_A$  and  $sk_A$  used for signature verification and generation, respectively.

B : a party involved in a fair exchange. It has a pair of public/private keys  $pk_B$  and  $sk_B$  used for signature verification and generation, respectively.

T :an off-line trusted third party (TTP). It has a pair of public/private keys PKT and SKT used for encryption and decryption, respectively

Remarks: the above keys of each parties are long term keys. There must be a secure binding between a party's identity and its public key. Such a binding may be in the form of a public key certificate issued by a certification authority. For references on public key encryption schemes, digital signature



schemes, encryption and decryption and one-way hash functions, public key certificates, see D. E. R. Denning, Cryptography and Data Security, Addition-Wesley, Reading, MA, 1983; W. Stallings, Network and Internetworks Security - Principles and Practice, Prentice Hall, Englewood Cliffs, NJ, 1995; and C. Kaufman, R. Perlman and M. Speciner, Network Security - Private Communication in a Public World, PTR Prentice Hall, Englewood Cliffs, NJ, 1995.

We will describe three protocols for fair exchange of digital data between distrustful parties A and B with an off-line trusted third party T. In all the protocols, we implement a new cryptographic mechanism called Certificate of Encrypted Message Being a Signature (CEMBS). A CEMBS is generated by the party who initiates a fair exchange to prove to others, in particular the other party, that an encrypted message is a certain party's signature on a known file while without revealing the signature. Let  $PKX/SKX$  be party X's public/private key pair in a public key encryption scheme and  $pkY/skY$  be party Y's public/private key pair in a digital signature scheme. Let  $sign\_Y = Ssign(skY, M)$  be Y's signature on a file M under  $skY$  and  $C\_X = Pencyr(PKX, sign\_Y)$  be the ciphertext of the encrypted signature  $sign\_Y$  under X's public key  $PKX$ . Party Y can generate a CEMBS, denoted as  $Cert\_Y\_X$ , to prove that  $C\_X$  is indeed the encryption (under  $PKX$ ) of the signature  $sign\_Y$  on M while without disclosing the signature. The  $Cert\_Y\_X$  can be verified by anyone using a public verification algorithm  $Veri$ , which on inputs  $Cert\_Y\_X, C\_X, M, PKX$ , and  $pkY$ , output "yes" or "no".

The CEMBS can be realized on cryptosystems with  $P = \text{ElGamal}$  public key encryption scheme and  $S = \text{DSA-like digital signature}$  scheme. It can also be realized on cryptosystems with  $P = \text{ElGamal}$  public key encryption scheme and  $S = \text{Guillou-Quisquater}$  digital signature scheme. Procedures on the realization and verification of CEMBA will be shown later.

## 1. Protocol 1 - Fair Exchange of Digital Signatures on A Common File

It is assumed that A and B have agreed on a common file (such as a digital contract document) M. Referring to Figure 1, the steps for A and B to exchange their digital signatures sign\_A and sign\_B on M are:

a. Party A, in step 100 using a Signature-Ciphertext-CEMBS-Generation Program (SCCGP), computes its signature  $\text{sign\_A} = \text{Ssign}(\text{skA}, M)$  on the file M, the ciphertext  $C\_T = \text{Pencr}(\text{PKT}, \text{sign\_A})$  on  $\text{sign\_A}$  under T's public key PKT, and the CEMBS  $\text{Cert\_A\_T}$  which is used to prove that  $C\_T$  is a ciphertext of  $\text{sign\_A}$  without disclosing the signature. A sends  $\text{MSG1} = (C\_T, \text{Cert\_A\_T})$  to B.

b. Party B, upon receiving MSG1 in step 120, checks whether  $\text{Veri}(\text{Cert\_A\_T}, \text{C\_T}, \text{M}, \text{PKT}, \text{pkA}) = \text{yes}$  in step 140. If the answer is "no", B does nothing or sends an alert signal to A in step 160; if it is "yes", B computes and sends his signature  $\text{sign B} = \text{S sign}(\text{skB}, \text{M})$  as MSG2 to A in step 180.

c. In step 200, A checks to see if it receives MSG2 and if so, checks whether  $S_{\text{veri}}(\text{pk}_B, \text{sign}_B, M) = \text{yes}$ . If A does not receive MSG2 or the received  $\text{sign}_B$  is not valid, A does nothing or sets up an alert signal to itself and B in step 220. If  $\text{sign}_B$  is valid, A accepts it and sends  $\text{sign}_A$  as MSG3 to B in step 240. At this point, A considers the fair exchange completed.

d. In step 260, B checks to see if it receives MSG3 and if

e. Upon receiving MSG4 in step 320, T in step 340 first checks sign\_B using B's public key pkB to make sure that it is B's signature on M. If sign\_B is correct, T decrypts C\_T using its private key SKT to get sign\_A and then checks whether it is A's signature on M using A's public key pkA. If both sign\_A and sign\_B are valid, T sends sign\_A in MSG5 to B and sign\_B in MSG6 to A in step 360. On the other hand, if either sign\_B or sign\_A is incorrect, T does nothing or send an alert signal to B in step 380.

g. Upon receiving MSG6 in step 420, A accepts sign\_B if it has not been accepted in step 240; otherwise, A discards MSG6.

It is apparent that if A and B both behave properly, they will obtain each other's signatures without any involvement of T. Now consider what happens if B performs improperly. B has two chances to perform improperly. The first one is in step 180 where B may send A an incorrect `sign_B`, but A can detect this in step 200 and refuse to give `sign A` to B. The second chance

## 2. Protocol 2 - Fair Exchange of Digital Signatures on Different Files

## 2. Protocol 2 - Fair Exchange of Digital Signatures on Different Files

Here we assume that A and B have agreed on two files  $M_A$  and  $M_B$ . The process for A and B to exchange their digital signatures on  $M_A$  and  $M_B$ , respectively, are identical to those in Protocol 1 except that 1) A's signature is on " $M_A || h(M_B)$ " and B's signature is on " $M_B || h(M_A)$ ", i. e.,  $sign_A = Ssign(sk_A, M_A || h(M_B))$  and  $sign_B = Ssign(sk_B, M_B || h(M_A))$ , where  $h()$  is a one-way hash function; 2) when B asks T's help in step 300, B sends  $M_A$ ,  $M_B$ ,  $C_T$ , and  $sign_B$  as MSG4 to T; and 3) upon receiving MSG4 in step 320, T in step 340 decrypts  $C_T$  to get sign A and checks to see if sign A and sign\_B are

### 3. Protocol 3 - Fair Exchange of Confidential Data and Signature

Figure 2 shows the process of exchanging a confidential message and a signature on the message between A and B. More specifically, this protocol lets A send a digital signature on a one-way hash  $h(M)$  of a file  $M$  to B in exchange for  $M$  from B. Note that A's signature is on  $h(M)$  instead of  $M$ . It is impossible for A to sign directly on  $M$  before A sees it. On the other hand, after A sees  $M$ , it may refuse to send B the signature. No protocol can solve this dilemma. To avoid A signing on  $h(M)$  but receives a message  $M'$  different from the desired  $M$ , we assume that A has means of obtaining a one-way hash of the desired message  $M$  in authenticated manners. As pointed out in M. K. Franklin and M. K. Reiter, "Fair exchange with a semi-trusted third party", Proceedings of the 4th ACM Conferences on Computer and Communications Security, pp. 1-5, April 1-4, 1997, Zurich, Switzerland, this assumption is justified in protocols and applications in which one-party is responsible for revealing the input that produces a known output, already validated as part of the protocol or application, from a one-way hash function. Examples include the S/KEY user authentication system, see N. M. Haller, "The S/KEY one-time password system", Proceedings of the Internet Society Symposium on Network and Distributed Systems, 1994, the PayWord

The steps of the exchanges are:

- c. In step 600, A checks to see if it receives MSG2 = M and if so, checks whether the one-way hash of the received message matches the known  $h(M)$ . If A does not receive MSG2 or M is not valid (i. e., the one-way hash of the received message does not match  $h(M)$ ), A does nothing or sets up an alert signal to itself and B in step 620. If the received M is valid, A accepts it and sends sign A in MSG3 to B in step 640. At this point, A

considers the fair exchange process completed.

d. In step 660, B checks to see if it receives MSG3 and if so, checks whether  $\text{Sveri}(\text{pkA}, \text{sign\_A}, h(M)) = \text{yes}$ . If B receives MSG3 and sign\_A is valid, it accepts sign\_A in step 680. At this point, B considers the fair exchange process completed. If B does not receive MSG3 or the received sign\_A is not valid, B sends M and C T to T in MSG4 in step 700.

e. Upon receiving MSG4 in step 720, T in step 740 first computes  $h(M)$  of the received M, decrypts  $C_T$  using its private key SKT to get  $sign\_A$  and then checks whether it is A's correct signature on  $h(M)$  using A's public key pkA. If it is, T sends  $sign\_A$  in MSG5 to B and sends M in MSG6 to A in step 760. On the other hand, if  $sign\_A$  is not a signature on the newly computed  $h(M)$ , T does nothing or send an alert signal to B in step 780.

f. Upon receiving MSG5 in step 800, B accepts sign\_A and terminates the session.

g. Upon receiving MSG6 in step 820, A accepts M if it has not been accepted in step 640; otherwise, A discards MSG6.

#### 4. The First Embodiment of the SCCGP

Figure 3 shows the flow chart of the first embodiment of the Signature-Ciphertext-CEMBS-Generation Program (SCCGP). It is



Let  $p$  and  $q$  be prime integers such that  $p = 2q + 1$ . For security reason, we require that  $q - 1$  have no small prime factors except 2. Let  $G$ , an element in  $\mathbb{Z}_p^*$ , have order  $q$  and  $g$  be a generator of  $\mathbb{Z}_q^*$ . We have

$$\text{PKT: } g^{\text{SKT}} \bmod q$$
$$\text{pkA: } G^{\text{skA}} \bmod p$$

Party A's signature on  $M$  under  $sk_A$  is  $S_{\text{sign}}(sk_A, M) = (r, s)$   
 where  $r = G^k \bmod p$  for an random element  $k$  in  $\mathbb{Z}_q^*$  and  $s =$

$(h(M) + r(sk_A)) / k \bmod q$ . Here  $h()$  is a one-way hash function. The verification  $S_{\text{veri}}(pk_A, (r, s), M)$  is to check whether  $r^s = (G^{h(M)})(pk_A^r) \bmod p$ .

CEMBS in the cryptosystem described above can be realized through Stadler's PEDLDLL (Proof of Equivalence of Discrete Logarithm to Discrete LogLogarithm), see M. Stadler, "Publicly verifiable secret sharing", Proceedings of Eurocrypt'96, LNCS 1070, Springer-Verlag, pp.190-199, 1996. The PEDLDLL problem is stated as following:

Let  $p$  and  $q$  be as defined above. Let  $x, y$  and  $z$  be elements in  $Z_q^*$  and  $X$  and  $Y$  be elements in  $Z_p^*$  where the order of  $X$  is  $q$ . There exists a  $a$  in  $\{1, 2, \dots, q-2\}$  such that  $y = x^a \bmod q$  and  $Y = X^{(z^a)} \bmod p$ . A prover, who knows  $a$ , can produce a PEDLDLL certificate to prove to a verifier that indeed  $y = x^a \bmod q$  and  $Y = X^{(z^a)} \bmod p$  for some  $a$  while not revealing  $a$  and  $z^a$ . Here  $x, y, z, X$ , and  $Y$  can be regarded as public values to the verifier.

The CEMBS  $\text{Cert\_A\_T}$  can be induced from a PEDLDLL certificate as follows. When party A encrypts the signature  $\text{sign\_A} = (r, s)$  under PKT, it only encrypts  $s$  while leaves  $r$  in plain. That is, the encrypted signature is  $C_T = (r, \text{Pencr}(\text{PKT}, s))$  where  $\text{Pencr}(\text{PKT}, s) = (W, VT)$ , with  $W = g^w$  and  $VT = s((\text{PKT})^w)$ . Hence, the encrypted message is A's signature on  $M$  implies that

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2110	2111	2112	2113	2114	2115	2116	2117	2118	2119	2120	2121
2122	2123	2124	2125	2126	2127	2128	2129	2130	2131	2132	2133
2134	2135	2136	2137	2138	2139	2140	2141	2142	2143	2144	2145
2146	2147	2148	2149	2150	2151	2152	2153	2154	2155	2156	2157
2158	2159	2160	2161	2162	2163	2164	2165	2166	2167	2168	2169
2170	2171	2172	2173	2174	2175	2176	2177	2178	2179	2180	2181
2182	2183	2184	2185	2186	2187	2188	2189	2190	2191	2192	2193
2194	2195	2196	2197	2198	2199	2200	2201	2202	2203	2204	2205
2206	2207	2208	2209	2210	2211	2212	2213	2214	2215	2216	2217
2218	2219	2220	2221	2222	2223	2224	2225	2226	2227	2228	2229
2230	2231	2232	2233	2234	2235	2236	2237	2238	2239	2240	2241
2242	2243	2244	2245	2246	2247	2248	2249	2250	2251	2252	2253
2254	2255	2256	2257	2258	2259	2260	2261	2262	2263	2264	2265
2266	2267	2268	2269	2270	2271	2272	2273	2274	2275	2276	2277
2278	2279	2280	2281	2282	2283	2284	2285	2286	2287	2288	2289
2290	2291	2292	2293	2294	2295	2296	2297	2298	2299	2300	2301
2302	2303	2304	2305	2306	2307	2308	2309	2310	2311	2312	2313
2314	2315	2316	2317	2318	2319	2320	2321	2322	2323	2324	2325
2326	2327	2328	2329	2330	2331	2332	2333	2334	2335	2336	2337
2338	2339	2340	2341	2342	2343	2344	2345	2346	2347	2348	2349
2350	2351	2352	2353	2354	2355	2356	2357	2358	2359	2360	2361
2362	2363	2364	2365	2366	2367	2368	2369	2370	2371	2372	2373
2374	2375	2376	2377	2378	2379	2380	2381	2382	2383	2384	2385
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The cryptosystem requires a trusted authorized center AC to create system parameters. AC chooses two primes  $R$  and  $Q$  where  $R = 2p'q+1$ ,  $Q = 2pq+1$  for primes  $p'$ ,  $p$  and  $q$ , sets  $n = RQ$  and chooses an element  $g$  in  $Z_n^*$  such that it has order  $q$ . Next, AC randomly chooses a large number  $v$  co-prime to  $(R-1)(Q-1)$  and publishes system parameters  $n$ ,  $g$ ,  $q$ ,  $v$ .  $R$  and  $Q$  can be destroyed and AC may cease to exist after this system initialization.

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P:      ElGamal system on  $(\mathbb{Z}_n^*, g)$ 
SKT:    randomly selected from  $\{1, 2, \dots, q-2\}$ 
PKT:     $g^{\text{SKT}} \bmod n$ 

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The ciphertext of  $m$ , an element in  $\mathbb{Z}_n^*$ , under PKT is  $\text{Pencr}(\text{PKT}, m) = (W, V)$ , where  $W = g^w \bmod n$  for a random  $w$  in  $\{1, 2, \dots, q-1\}$  and  $V = m(\text{PKT})^w \bmod n$ . The decryption is  $m = V/W^{\text{SKT}} \bmod n$ . Further, we have



d. Output sign\_A, C\_T, and Cert\_A\_T in step 1280.

Note that verification of Cert\_A\_T is to check whether  $c = H(g || W || PKT^v || (VT^v)/V || (g^r)(W^c) || ((PKT^v)^r)((VT^v/V)^c))$  holds true.

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